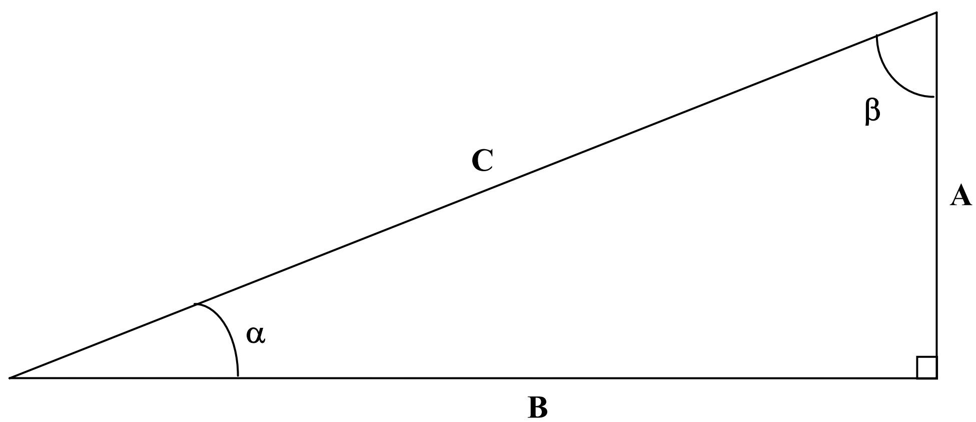
Trigonometry

It is possible to solve many force and velocity problems by drawing vector diagrams. However, the degree of accuracy is dependent upon the exactness of the person doing the drawing and measuring. In addition, this approach is time consuming compared to the quicker and more accurate method of using trigonometry.

The word **trigonometry** literally means, "*the measurement of triangles*". Many problems in motion analysis involve the use of right triangles. A right triangle is one containing an internal right angle (90º). It should be recalled that **the sum of the three internal angles of a triangle is equal to 180**º. Also, angles less than 90º are called **acute** angles, while those greater than 90º are called **obtuse** angles.

Two angles are said to be **complementary** if their sum equals 90º. So, in a right triangle, it should be noted that the two acute angles are complementary. If the sum of two angles equals 180º, the angles are said to be **supplementary**.

In order to understand the trigonometric functions, one must first be able to identify the parts of a right triangle. The diagram below will be used for the purpose of explanation.



The six component parts of the right triangle consist of three angles and three sides. It should be noted that the longest of the three sides (C) is opposite to the right angle. The longest side is called the **hypotenuse**. Since the hypotenuse is always opposite the right angle, it is very easy to identify. The other two sides are referred to as the legs of the triangle. Two notations are typically given to the legs: the **opposite side**, and the **adjacent side**. How these legs are defined is strictly dependent upon the question at hand. Referring to the diagram, the side opposite angle α is side A. On the other hand, the side opposite to angle β is side B. The side adjacent to angle α would be side B.

The sides of a triangle can be used to represent distance, magnitude of force, velocity, or some other physical property. However, before we can use such techniques, we need to understand the fundamental trigonometric and geometric properties associated with the right triangle.

An easy way to remember trigonometric properties is: “**SOH CAH TOA**”

SOH = "**S**ine of the angle= $\dfrac{\text{Opposite side}}{\text{Hypotenuse}}$

CAH = "**C**osine of the angle = $\dfrac{\text{Adjacent side}}{\text{Hypotenuse}}$

TOA = "**T**angent of the angle = $\dfrac{\text{Opposite side}}{\text{Adjacent side}}$

If we know the length of the sides of a right triangle, we can compute the internal angles. We can simply use the *inverse* of the trigonometric properties we defined above. That is, we can use the **ARCSIN ($\sin^{-1}$)**, **ARCCOS ($\cos^{-1}$)**, and **ARCTAN ($\tan^{-1}$)** functions.

ARCSIN: *"arc sine" = "the actual angle whose sine equals Opp / Hyp"*
ARCCOS: *"arc cosine" = "the actual angle whose cosine equals Adj / Hyp"*
ARCTAN: *"arc tangent" = "the actual angle whose tangent equals Opp / Adj"*

Example:

Side A = 10 inches
Side B = 3 inches
Find angle α (in degrees)

We know:

$$\text{TOA} = \text{Tangent of } \alpha = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{\text{Side A}}{\text{Side B}} = \frac{10''}{3''}$$

So, $\tan \alpha = 10 / 3 = 3.333$; but that is not the actual value of the angle α.

$\alpha = \tan^{-1}(3.333)$

$\alpha = 73.3$ degrees

Also, the length of any side of a right triangle can be computed if the lengths of the other two sides are known. This property is derived from the **Pythagorean Theorem**.

$\mathbf{C^2 = A^2 + B^2}$

Example:

Referring to the diagram on the first page, assume that the hypotenuse is 6 inches long and angle $\alpha = 40^\circ$. Find the following:

A) Angle β
B) Side A
C) Side B

Angle β can be readily found since it is complementary to angle α. That is, $\alpha + \beta = 90^\circ$. So:

$$\beta = 90^\circ - \alpha$$
$$\beta = 90^\circ - 40^\circ$$
$$\beta = 50^\circ$$

To find side A, we must select a trigonometric function that involves side A (unknown), and side C, the hypotenuse (known). It can be seen that:

$$\sin\alpha = \frac{\text{Side A}}{\text{Side C}}$$ (remember: **SOH**)

We know that $\alpha = 40^\circ$, and Side C = 6 inches. We simply need to solve this equation for the unknown value of Side A.

$$\begin{aligned}\text{Side A} &= \sin\alpha \cdot \text{Side C}\\ &= \sin(40^\circ) \cdot 6''\\ &= 0.643 \cdot 6''\\ &= 3.857''\end{aligned}$$

Side B may be found through the same type of process. However, since "there is more than one way to skin a cat", let's use the Pythagorean Theorem.

We know that $\mathbf{C^2 = A^2 + B^2}$. So, if we solve for Side B, we can show:

$$B^2 = C^2 - A^2$$

$$B = \sqrt{C^2 - A^2}$$
$$B = \sqrt{6^2 - 3.857^2}$$
$$B = \sqrt{36 - 14.876}$$
$$B = \sqrt{21.124}$$

$$B = 4.596''$$

Problems: Use your knowledge of trigonometry to solve each of the problems below. (Hint: sketching the triangles is very helpful)

1. Given the hypotenuse = 10 m, angle $\alpha = 30^{\circ}$, find the length of both legs of the triangle.

2. Given angle $\alpha = 55^{\circ}$, opposite side = 4 m, find the hypotenuse and adjacent side lengths.

3. Given the hypotenuse = 4 m, side A = 3 m, find both acute angles.

4. Given the length of the side A = 3 m, and side B = 5 m, find the acute angles and the hypotenuse.

5. A ball is kicked into the air at an angle of 25° to the horizontal with an initial resultant velocity of 25 m/sec. Find both the vertical and horizontal components of the velocity vector.

6. At the instant of take-off, a long jumper has a forward velocity of 9.47 m/sec and a vertical velocity of 3.21 m/sec. Find the angle of take-off (relative to the horizontal) and the magnitude of the resultant velocity vector.

Practice Trigonometry Problems Solutions

1. $A = 10 \cdot \sin 30^{o} = 5$
   $B = 10 \cdot \cos 30^{o} = 8.66$

2. $\tan 55 = 4 \div A$
   $A = 4 \div \tan 55^{o}$
   $A = 2.8$
   $C = \sqrt{4^2 + 2.8^2} = 4.88$

3. $\cos \alpha = 3 \div 4$
   $\alpha = \cos^{-1}(0.75)$
   $\alpha = 41.4^{o}$
   $\beta = 90^{o} - 41.4^{o}$
   $\beta = 48.6^{o}$

4. $C = \sqrt{3^2 + 5^2} = 5.83$
   $\alpha = \tan^{-1}(5 \div 3) = 59.04^{o}$
   $\beta = 90^{\circ} - 59.04^{o} = 30.96^{o}$

5. $V_V = 25 \text{ ft/s} \cdot \sin 25^{o} = 10.57 \text{ ft/s}$
   $V_H = 25 \text{ ft/s} \cdot \cos 25^{o} = 22.66 \text{ ft/s}$

6. $\tan \alpha = 3.21/9.47$
   $\alpha = \tan^{-1}(0.339) = 18.72^{o}$

$V_{RESULTANT} = \sqrt{3.21^2 + 9.47^2} = 9.99 \text{ ft / s}$