Eliminate Iteration from Flow Problems

John D. Barry
Middough, Inc.

This article introduces a novel approach to solving flow and pipe-sizing problems based on two new dimensionless quantities that are independent of line size and flow.

Process engineers are often faced with the task of sizing a pipe for a specific flow. A common approach to this problem starts with a typical fluid velocity (*e.g.,* 5 ft/s for a liquid with properties roughly similar to water). Calculating the pressure drop across a line sized in this way is a straightforward matter, involving the friction factor and either the equivalent length or the sum of resistances to flow (the *K* values). This calculation is detailed in many standard references.

However, that is only one of three possible pipe sizing/flow problems an engineer is likely to encounter. The other two, which are encountered less commonly, are:

• given a required flow and pressure drop criterion, what line size is required?

• given a line size and a pressure drop, what flow may be expected? This is the conceptual inverse of the previous problem.

The solution of either of these problems typically requires an iterative approach. This article introduces a novel approach for the direct (non-iterative) solution of such problems.

Friction in pipe flow — the classic approach

The Bernoulli equation, also known as the mechanical energy balance, is the basis for understanding flow in pipes:

$$\Delta\frac{v^2}{2g_c}+\frac{g}{g_c}\Delta z+\frac{\Delta P}{\rho}+lw_f+W=0 \qquad (1)$$

The first term, accounting for kinetic energy changes, is usually small compared to the other four terms, and thus can be assumed to be negligible. Note that for line sizing purposes, the envelope for a mechanical energy balance typically excludes rotating equipment (*e.g.,* a pump or a turbine) and focuses on the terminal pressure conditions, elevation changes, and friction losses; the rotating equipment is then sized to meet the conditions derived from this mechanical energy balance.

The second term in the energy balance refers to elevation changes; those are typically defined as part of the conceptual layout of the proposed line. The third term, referring to the difference in terminal pressures, is also typically defined as part of the piping layout. This approach focuses on the fourth term, lw_f, the work lost due to friction:

$$lw_f=\frac{fLv^2}{2g_cD} \qquad (2)$$

In this equation, f is the Darcy friction factor:

$$f=\frac{2g_c(-\Delta P)D}{v^2\rho L} \qquad (3)$$

The Darcy friction factor equals four times the Fanning friction factor, $f_{Fanning}$. To use the Fanning friction factor, substitute $4f_{Fanning}$ for f wherever the latter appears.

The friction factor is a function of both the fluid Reynolds number, $Re = Dv\rho/\mu$, and the relative roughness, ε/D. The relationship between these quantities and the friction factor is expressed graphically in a Moody plot, or mathematically in various empirical relations (*e.g.,* the Colebrook or Churchill equations). To solve the types of problems considered here, it is necessary to introduce dimensionless quantities that do not depend on the line size, D, or the fluid velocity, v.

Eliminating the diameter

Bennett and Myers *(1)* suggest that a plot or correlation of the friction factor, f, as a function of $Ref^{1/5}$ would be useful in solving for line size if the flowrate and pressure drop are known. This dimensionless quantity ($Ref^{1/5}$) is drawn from two quantities already established, namely, the Reynolds number and the Darcy friction factor. The velocity, v, appears in both Re and f. It can be eliminated by using the definition of the velocity:

$$v = \frac{Q}{A} = \frac{Q}{\pi D^2/4} = \frac{4Q}{\pi D^2}$$

$$v^2 = \frac{16Q^2}{\pi^2 D^2}; \quad \frac{1}{v^2} = \frac{\pi^2 D^2}{16Q^2} \tag{4}$$

Thus, $Re = 4Q\rho/\pi D\mu$ and $f = [2g_c(-\Delta P)\,\pi^2 D^5] / [16Q^2\rho L] = [\pi^2 g_c(-\Delta P)D^5] / [8Q^2\rho L]$, which leads to:

$$\begin{aligned} \mathrm{Re}\, f^{1/5} &= \left(\frac{4Q\rho}{\pi D\mu}\right)\left[\left(\frac{\pi^2 g_c(-\Delta P)}{8Q^2\rho L}\right)D^5\right]^{1/5} \\ &= \left(\frac{4Q\rho}{\pi D\mu}\right) D\left(\frac{\pi^2 g_c(-\Delta P)}{8Q^2\rho L}\right)^{1/5} \\ &= \left(\frac{4Q\rho}{\pi\mu}\right)\left(\frac{g_c(-\Delta P)\pi^2}{8Q^2\rho L}\right)^{1/5} \end{aligned} \tag{5}$$

The diameter has now been eliminated. But f, and thus $Ref^{1/5}$, depends on the relative roughness, which presupposes knowledge of the pipe diameter. To get around this, a new dimensionless quantity, the flow function, is introduced:

$$\begin{aligned} \Theta = \mathrm{Re}\, f^{2/5}\frac{\varepsilon}{D} &= \left(\frac{4Q\rho}{\pi D\mu}\right)\left[\left(\frac{g_c(-\Delta P)\pi^2}{8Q^2\rho L}\right)D^5\right]^{2/5}\frac{\varepsilon}{D} \\ &= \left(\frac{4Q\rho}{\pi D\mu}\right) D^2\left(\frac{g_c(-\Delta P)\pi^2}{8Q^2\rho L}\right)^{2/5}\frac{\varepsilon}{D} \\ &= \left(\frac{4Q\rho\varepsilon}{\pi\mu}\right)\left(\frac{g_c(-\Delta P)\pi^2}{8Q^2\rho L}\right)^{2/5} \end{aligned}$$

$$\begin{aligned} \Theta = \mathrm{Re}\, f^{2/5}\frac{\varepsilon}{D} &= \left(\frac{4\varepsilon}{\pi\mu}\right)\left(\frac{g_c(-\Delta P)\pi^2}{8L}\right)^{2/5}\left(Q\rho^3\right)^{1/5} \\ &= \left(\frac{4\varepsilon}{\pi\mu}\right)\left(\frac{g_c(-\Delta P)\pi}{2L}\right)^{2/5} W^{1/5}\rho^{2/5} \end{aligned} \tag{6}$$

Nomenclature

- $a, b, c,\ldots$ = coefficients in correlating polynomial (Eq. 9) (represented by y in the general case of Eq. 10), dimensionless
- $A, B, C,\ldots$ = coefficients in generating polynomial (Eq. 10), dimensionless
- A = pipe cross-sectional area normal to flow, ft^2
- D = pipe (inside) diameter, ft
- f = Darcy friction factor, dimensionless
- g = acceleration of gravity, 32.174 ft/s^2
- g_c = conversion factor, 32.174 ft-lb/lb$_{force}$-s^2
- Ka = Kármán number (Eq. 7), dimensionless
- lw_f = lost work due to friction, ft-lb$_{force}$/lb
- L = line equivalent length, ft
- P = pressure, psi or lb$_{force}$/ft^2
- Q = volumetric flowrate, ft^2 /s
- Re = Reynolds number, dimensionless
- v = fluid velocity, ft/s
- W = shaft work, ft-lb$_{force}$/lb
- W = mass flowrate, lb/s
- y = generic coefficient in correlating polynomial (Eq. 9), generated by Eq. 10, dimensionless
- z = elevation, ft

Greek Letters

- ε = absolute roughness of pipe, ft
- Φ = friction/roughness function (Eq. 8), dimensionless
- μ = dynamic viscosity, lb/ft-s
- Θ = flow function (Eq. 6), dimensionless
- Θ^* = modified flow function = $Ref^{1/5}$, dimensionless
- ρ = density, lb/ft^2

which can be calculated from known quantities. The absolute roughness, ε, is a function of the nature of the pipe, which is known, and is independent of the line diameter.

A complementary dimensionless quantity that eliminates dependence on the flow is also needed. This has already been established in the form of the Kármán number, Ka:

$$\begin{aligned} Ka = \Lambda = \mathrm{Re}\sqrt{f} &= \left(\frac{D\rho v}{\mu}\right)\sqrt{\frac{2g_c(-\Delta P)D}{v^2\rho L}} \\ &= \frac{D\rho}{\mu}\sqrt{\frac{2lw_f g_c D}{L}} \\ &= \frac{1}{\mu}\sqrt{\frac{2g_c(-\Delta P)D^3\rho}{L}} \\ &= \sqrt{\frac{2g_c(-\Delta P)D^3\rho}{\mu^2 L}} \end{aligned} \tag{7}$$

Many texts provide plots of the friction factor as a function of the Kármán number with the relative roughness as a variable. This may not be particularly helpful, however, since it presumes knowledge of the pipe diameter, which may not be the case. To work around this, another new dimensionless quantity, the friction/roughness function, is defined:

$$\Phi = \left[f^{1/5}\left(\frac{\varepsilon}{D}\right)\right]^{-1} = \left[\left(\frac{\pi^2 g_c(-\Delta P)D^5}{8Q^2\rho L}\right)\left(\frac{\varepsilon}{D}\right)\right]^{-1}$$
$$= \left\{\left[\left(\frac{\pi^2 g_c(-\Delta P)}{8Q^2\rho L}\right)\right]^{1/5} D\left(\frac{\varepsilon}{D}\right)\right\}^{-1}$$
$$= \left\{\varepsilon\left[\left(\frac{\pi^2 g_c(-\Delta P)}{8Q^2\rho L}\right)\right]^{1/5}\right\}^{-1} \qquad (8)$$

Guidelines for sizing pipe (*e.g.,* Peters and Timmerhaus *(2)*) include typical velocities in the range of 3–10 ft/s for liquids and 50–150 ft/s for gases. A quick check of Reynolds numbers calculated using velocities within these ranges and properties of common industrial liquids (*e.g.,* organic liquids with viscosities of approximately 1 cP and densities on the same order of magnitude as that of water) shows that flows meeting these guidelines are indeed turbulent; similar comments apply for industrial gases. Most design courses guide the student to design for turbulent flow in pipelines, since this is perhaps the most common situation in industry.

Thus, it is reasonable to assume full turbulence and, as a corollary, define a slightly modified version of Φ to use the fully turbulent Darcy friction factor, f_T. This modified quantity Φ_T, is the product of $f_T^{1/5}$ and the relative roughness ε/D. This quantity will prove useful in one of the variations of the classic flow problem presented later.

For the calculation of flow based on pipe size and pressure drop, values of f_T as a function of line size or relative roughness are tabulated in the literature *(3)*, and are listed in Tables 1 and 2.

Now we have a dimensionless quantity, Θ, that is independent of line size that can be used to correlate another dimensionless quantity, *Ka*. This latter dimensionless quantity is independent of flow.

Table 1. Fully turbulent Darcy friction factor as a function of line size.

Line Size (nominal), in.	f_T	Line Size (nominal), in.	f_T
0.5	0.027	4	0.017
0.75	0.025	6	0.015
1	0.023	8–10	0.014
1.5	0.021	12–16	0.013
2	0.019	18–24	0.012
3	0.018		

Source: *(3)*.

Plotting log *Ka* as a function of log Θ and Θ_T yields a family of parallel curves. Multiple regression analysis shows that these curves can be represented by polynomials with log Θ as the independent variable and the form:

$$\log Ka = a + b(\log\Theta) + c(\log\Theta)^2 + d(\log\Theta)^3 + \ldots \qquad (9)$$

We'll call this relationship the correlating polynomial. A cubic polynomial is generally sufficient to correlate log *Ka* as a function of log Θ (for reasons that will be discussed later). Multiple regression analysis shows that *a, b, c, d* … are well-correlated by polynomials in log Φ_T. Thus:

$$y = A + B(\log\Phi_T) + C(\log\Phi_T)^2 + D(\log\Phi_T)^3 + E(\log\Phi_T)^4 \qquad (10)$$

where *y* is a generic coefficient for Eq. 9. We'll call this relationship the generating polynomial to distinguish it from the correlating polynomial that relates log *Ka* to log Θ. A quartic (fourth-degree) polynomial is sufficiently accurate for most work. The coefficients *A, B, C, D* and *E* for Eq. 10 are given in Table 3.

Knowing the physical definition of the system (line [equivalent] length, pressure drop, nature of the pipe),

Table 2. Fully turbulent Darcy friction factor as a function of relative roughness.

ε/D	f_T	ε/D	f_T
0.07	0.084201	0.0003	0.014937
0.06	0.078021	0.0002	0.01373
0.05	0.071551	0.00015	0.01296
0.04	0.064671	0.0001	0.01198
0.03	0.057174	0.00009	0.011743
0.02	0.048637	0.00008	0.011487
0.015	0.04369	0.00007	0.011207
0.01	0.037904	0.00006	0.010896
0.009	0.036588	0.00005	0.010544
0.008	0.035197	0.00004	0.010137
0.007	0.033714	0.00003	0.009645
0.006	0.032116	0.00002	0.009011
0.005	0.030367	0.000015	0.008598
0.004	0.028416	0.00001	0.008063
0.003	0.026165	0.000009	0.007932
0.002	0.02342	0.000008	0.00779
0.0015	0.021727	0.000007	0.007633
0.001	0.019635	0.000006	0.007457
0.0009	0.019141	0.000005	0.007257
0.0008	0.018611	0.000004	0.007023
0.0007	0.018036	0.000003	0.006738
0.0006	0.017404	0.000002	0.006365
0.0005	0.016699	0.0000015	0.006119
0.0004	0.015893	0.000001	0.005795

Note: These values are calculated from the Churchill relationship for the friction factor using a Reynolds number contribution of zero.

Table 3. Generating polynomial coefficients (for use in calculating correlating polynomial coefficients).

	A	B	C	D	E
Quadratic					
a	–0.17163319	0.86401247	0.02040069	–0.00265645	0.00015052
b	1.02119400	–0.01740500	0.00185105	0.00005060	–0.00000845
c	–0.00121903	0.00082246	0.00002517	–000000668	–0.00000046
Cubic					
a	–0.30973737	1.04093058	–0.05082539	0.00973526	–0.00048466
b	1.10906642	–0.11763886	0.03482046	–0.00418687	0.00018018
c	–0.01381511	0.01267707	–0.00223295	0.00004726	00000891
d	0.00042345	–0.00022592	–0.00008810	0.00003103	–0.00000224
Correlating polynomial coefficients, smooth pipe					
Quadratic	*a*	*b*	*c*		
	–0.0920095	0.8881693	0.0046269		
Cubic	*a*	*b*	*c*	*d*	
	0.09043103	0.79275882	0.01945720	–0.00070362	

Table 4. Data for Example 1.

Given Data
Density, ρ = 54.7 lb/ft^3
Viscosity, μ = 1.8 cP
Line Length, L = 12,000 ft
Flowrate, Q = 1,750 gal/min
Absolute Roughness, ε = 0.00015 ft *(3)*

Derived Data
Pressure Drop Between Tie-In Point and Maximum Downstream Terminal Pressure, $-\Delta P$ = 75 psi

the flowrate, and the fluid's physical properties (density, viscosity) allows Θ, log Θ, Φ_T and log Φ_T to be determined. Log Φ_T can be used to calculate the coefficients *a, b, c* and *d* (via Eq. 10) and the tabulated values for *A, B, C, D* and *E*, which are used with Eq. 9 to yield log *Ka* and therefore *Ka*. Finally, knowing *Ka* leads directly to a theoretical line size:

$$Ka = \sqrt{\frac{2g_c(-\Delta P)D^3\rho}{\mu^2 L}}$$

$$Ka^2 = \frac{2g_c(-\Delta P)D^3\rho}{\mu^2 L}$$

$$D^3 = \left(\frac{\mu^2 Ka^2}{\rho}\right)\left(\frac{L}{2g_c(-\Delta P)}\right)$$

$$D = \left[\left(\frac{\mu^2 Ka^2}{\rho}\right)\left(\frac{L}{2g_c(-\Delta P)}\right)\right]^{1/3} \qquad (11)$$

It is unlikely that the diameter calculated in this way will be equal to a commercial pipe size. In most cases, the pressure drop specification is a maximum allowable pressure drop, so the next larger pipe size should be chosen. (Conversely, if a minimum pressure drop were specified, the next smaller line size would be chosen.)

Example 1

A refinery needs to move 60,000 bbl/d (1,750 gal/min) of a product with an API specific gravity of 30 (approximately 54.7 lb/ft^3) and a viscosity of 1.8 cP. The pressure at the new tie-in point (*i.e.,* the source) for this new line is 90 psig. Due to the design of existing equipment, the discharge pressure downstream cannot exceed 15 psig. The proposed line routing (approximately 12,000 equivalent ft, accounting for fittings) is essentially flat. Refinery specifications call for carbon steel pipe to be used. What size pipe is needed?

Solution. The data for the problem are given in Table 4. The calculations are detailed in the box on the next page.

1. Calculate Θ using Eq. 6, and then log Θ. Θ = 9.562, log Θ = 0.9805.

2. Calculate Φ using Eq. 8, and then log Φ. Φ = 12,511, log Φ = 4.0973.

3. Use the generating polynomial, Eq. 10, to get coefficients for the *Ka*-Θ relationship (for simplicity, use the quadratic form). These coefficients are a = 3.5707, b = 1.0363, and c = –0.00058.

4. Use these coefficients in Eq. 9 and the value of Θ calculated in Step 1 to generate *Ka*: log *Ka* = 3.5707 + (1.0363)(0.9805) + (–0.00058)(0.9805)^2 = 4.5862. So, *Ka* = 38,569.89054.

5. Solve for D using Eq. 11. D = 0.882 ft = 10.589 in.

Since a minimum pressure drop was specified, the next-smaller commercially available line size would be chosen to ensure that this minimum drop criterion is met. Thus, a 10-in. line (actual ID = 10.02 in. for Sch. 40 pipe) should be selected.

Estimating flow

This approach can also be used to calculate the expected flow through a pipe of a specified size with a known pressure drop. The terms used to calculate *Ka* (and thus log *Ka*) are known. Again fully turbulent flow is assumed, since that is common in industry. Then ε/D can be calculated based on the known line size and the nature of the pipe, and f_T can be obtained from the tabulated values as mentioned previously. That allows Φ_T to be calculated, as well as the coefficients *a, b, c* and *d* for the correlating polynomial relating log *Ka* to log Θ.

Article continues on next page

Calculations for Example 1

Step 1:

$$\Theta = \left(\frac{4(0.00015\text{ ft})}{\pi(1.8\text{ cP})\left(\frac{6.72\times10^{-4}\text{ ft-lb/s}}{\text{cP}}\right)}\right)\left[\frac{\left(\frac{32.174\text{ ft-lb}}{\text{lb}_f\text{-s}^2}\right)(75\text{ psi})\left(\frac{144\text{ in}^2}{\text{ft}^2}\right)\pi^2}{8(12{,}000\text{ ft})}\right]\left[\left(\frac{1{,}750\text{ gal}}{\text{min}}\right)\left(\frac{0.002223\text{ ft}^3/\text{s}}{\text{gal/min}}\right)\left(\frac{54.7\text{lb}}{\text{ft}^3}\right)\right]^{1/5} = 9.562$$

Step 2:

$$\Phi = \left\{\left[\left(\frac{2\left(\frac{32.174\text{ ft-lb}}{\text{lb}_f\text{-s}^2}\right)(75\text{ psi})\left(\frac{144\text{ in}^2}{\text{ft}^2}\right)}{8\left[\left(\frac{1{,}750\text{ gal}}{\text{min}}\right)\left(\frac{0.002223\text{ ft}^3/\text{s}}{\text{gal/min}}\right)\right]^2\left(\frac{54.7\text{lb}}{\text{ft}^3}\right)(12{,}000\text{ ft})}\right)\right]^{1/5}(0.00015\text{ ft})\right\}^{-1} = 12{,}511.14$$

Step 5:

$$D = \left[\left(\frac{\left[(1.8\text{ cP})\left(6.72\times10^{-4}\frac{\text{ft-lb/s}}{\text{cP}}\right)\right]^2(38{,}569.891)^2}{\left(54.7\frac{\text{lb}}{\text{ft}^3}\right)}\right)\left(\frac{(12{,}000\text{ ft})}{2\left(32.174\frac{\text{ft-lb}}{\text{lb}_f\text{-s}^2}\right)\left((75\text{ psi})\left(144\frac{\text{in}^2}{\text{ft}^2}\right)\right)}\right)\right]^{1/3} = 0.882\text{ ft} = 10.589\text{ in.}$$

This new approach uses the fully turbulent friction factor for a given line size as a contributing parameter to the generating equation (Eq. 10). The results of the generating equation are ultimately employed to get a value of Θ and thus the flow. One could then re-evaluate the friction factor based on this flow and the known line size and use the generating equation again, but it's up to the user to evaluate whether this would have practical value.

In the traditional approach (specifying physical properties and terminal conditions), one may calculate flow directly by postulating complete turbulence in addition to the known constraints (pressure drop, line length, physical properties). In that case, it is necessary to verify that the calculated flow does indeed yield a Reynolds number that qualifies as fully turbulent. Again, further refinement by iteration would be at the user's discretion.

It was suggested previously that a cubic polynomial should suffice to relate log *Ka* to log Θ, based on the observation that a cubic polynomial is the highest order equation that can relatively easily be solved analytically. (It is possible to solve a quartic polynomial analytically, but the solution is considerably more involved than solving a cubic polynomial. There is no general analytic solution for higher-order polynomials — and it's unlikely that there would be much, if any, benefit to using one.) Methods of solving cubic and quadratic equations are described in detail at http://mathworld.wolfram.com *(4)*.

Of the three roots arising from the solution of a cubic equation, only one is of interest. Values of log ? within the scope of this correlation fall within the approximate range of –3.5 ≤ log Θ ≤ 7.5.

With log Θ and thus Θ known, a flowrate can be found by solving Eq. 6:

$$\left(Q\rho^3\right)^{1/5} = \left(\frac{\pi\mu\Theta}{4\varepsilon}\right)\left(\frac{8L}{g_c(-\Delta P)\pi^2}\right)^{2/5}$$

$$Q = \left(\frac{1}{\rho^3}\right)\left[\left(\frac{\pi\mu\Theta}{4\varepsilon}\right)\left(\frac{8L}{g_c(-\Delta P)\pi^2}\right)^{2/5}\right]^5 \quad (12)$$

$$\left(W\rho^2\right)^{1/5} = \left(\frac{\pi\mu\Theta}{4\varepsilon}\right)\left(\frac{8L}{g_c(-\Delta P)\pi^2}\right)^{2/5}$$

$$W = \left(\frac{1}{\rho^2}\right)\left[\left(\frac{\pi\mu\Theta}{4\varepsilon}\right)\left(\frac{8L}{g_c(-\Delta P)\pi^2}\right)^{2/5}\right]^5 \quad (13)$$

Example 2

Chilled water at 45°F flows from a constant-level reservoir through a 2-in. Sch. 40 steel pipe, the end of which is open to the atmosphere. The pipe has an equivalent length of 175 ft, and the outlet is 35 ft below the liquid level in

Table 5. Data for Example 2.
Given Data
Density, ρ = 62.42 lb/ft^3
Viscosity, μ = 1.417 cP
Line Length, L = 175 ft
Pipe ID, D = 2.067 in.
Absolute Roughness, ε = 0.00015 ft *(3)*
Derived Data
Head Change, $-\Delta P$ = 35 ft liquid = 15.15 psi
Relative Roughness, ε/D = 0.000871
Fully Turbulent Darcy Friction Factor, f_T = 0.019 *(3)*

the reservoir. Neglecting any kinetic energy contributions, determine the flow.

Solution. The data for this problem appear in Table 5. In the interest of space, the calculations are not detailed here.

1. Using Eq. 7, calculate Ka = 16,801.2. Then, $\log Ka$ = 4.2253.

2. Since the nature of the pipe and the line size are known, ε/D, as well as the fully turbulent friction factor for that relative roughness, are also known. Therefore, Φ is obtained from its definition, Eq. 8: $\Phi = [f^{1/5}(\varepsilon/D)]^{-1}$ = 2,536.981. So, $\log \Phi$ = 3.404317.

3. Use Φ and the generating polynomial (Eq. 10) to calculate the coefficients a, b and c of the correlating polynomial (Eq. 9). For the purpose of illustration, use a quadratic generating polynomial. Thus: a = 2.9216, b = 0.970, and c = 0.002223.

4. Solve the quadratic correlating polynomial for Θ:

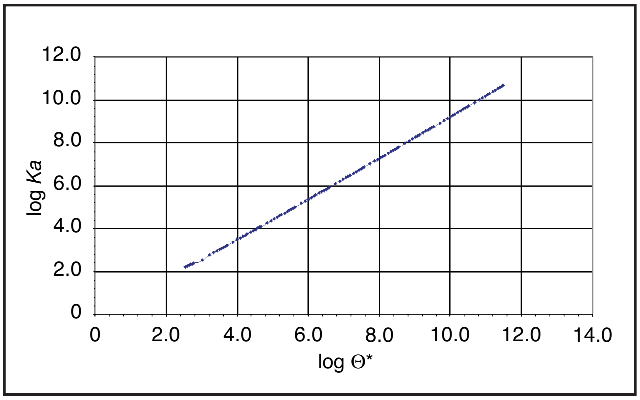
$$x = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}$$

where $\beta = b = 0.970$, $\gamma = a - \log Ka = -1.3034$, $\alpha = c$ = 0.002223. Thus, $x = \log \Theta$ = 1.340 and –437.766. The first root reflects the use of the positive sign of the radical; the second root, the negative sign. Only the first root has any physical significance, so with $\log \Theta$ = 1.340, Θ = 21.878.

5. Use Eq. 12 to derive the flowrate: Q = 0.259 ft^3/s = 116.4 gal/min.

The special case of smooth pipe

By definition, smooth pipe has zero roughness, which would render the definition of Θ useless. However, Gilmont's work *(5)* can be modified to correlate Ka as a function of Θ^*, where $\Theta^* = Ref^{1/5}$, to obtain a relationship where one variable is independent of the line size and the other is independent of the flowrate. Like the Θ-Ka correlation, this Θ^*-Ka correlation can be represented well by a cubic polynomial. Therefore, the techniques discussed above for rough pipe can be applied to smooth pipe using the data at the bottom of Table 3 and the relationship in Figure 1.



■ Figure 1. Kármán number as a function of Q^* for smooth pipe (ε = 0).

A final note

Line sizing is not exact or rigorous, since it involves discrete standard commercial sizes rather than values of a continuous function. Calculations may indicate that a pipe diameter of, say, 5.4 in., is required to accommodate a given flow, but that's simply the solution to an equation. Rather, one chooses the closest commercial size to suit the application based on the theoretical results of calculations, engineering judgment, and experience. CEP

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JOHN D. BARRY, P.E., is a senior process engineer with Middough, Inc. (Mt. Laurel, NJ; E-mail: john.barry.engr@comcast.net). Previously, he worked for several engineering firms in the greater Philadelphia area, as well as performed contract engineering work for DuPont (Deepwater, NJ and Wilmington, DE) Premcor (now Valero; Delaware City, DE).He holds a BS in chemistry from the Univ. of Delaware, and an MS in chemical engineering from the Univ. of Maryland. He is a professional engineer licensed in New Jersey and Delaware, and is a member of AIChE.